**WHAT**

Inductive reasoning involves students making careful observations and using concrete evidence to form plausible solutions and conclusions. Through keener observation and stronger evidence, inductive reasoning helps establish a more accurate conclusion.

In deductive reasoning, one makes use of axioms, proven laws and first principles to reach a conclusion. In contrast, inductive reasoning begins with observable phenomena and the use of experiments to build a hypothesis for a solution. In a sense, inductive reasoning does not allow one to make watertight conclusions (Hawthorne, 2012). However, with further experiments and continuous improvements made to the working hypothesis, a conclusion of greater strength can be achieved. This promotes a spirit of calculated risk-taking and data-driven decision making as the learner is constantly building on prior observations to arrive at an informed conclusion.

Most of us may have little, if any, memories of how we first started learning. However, if we observe how an infant learns, it is clear that it is through a process of close observations, mimicry and extensive trial-and-error. In other words, most of us began learning through inductive reasoning.

**Relevance to the Maths Curriculum**

Around 300BC, Euclid’s Elements was widely believed to have answered geometry’s most challenging problems and the theorems were deductively proven (Stamper, 1909). Today, many mathematics classes still make use of deductive approaches with great effectiveness. This approach is conclusive, efficient and remains a powerful method for logical thinking. However, the challenge is that deductive lessons may not always provide the necessary context needed for understanding abstract concepts.

Without meaningful perspectives and context, students may memorise theories and formulae without underlying foundational understanding. For example, in teaching geometrical concepts, a teacher might begin with the formula for calculating the hypotenuse. Using this proven formula, students then practise applying it on various right-angled triangles. Upon doing so successfully, the learners are content that they understand Pythagoras’ Theorem. There are benefits to this approach, but it also compromises valuable learning opportunities for students to observe a phenomenon and arrive at a conclusion on their own.

Through inductive reasoning, students can construct their own knowledge by observing, hypothesising, testing their hypotheses, and reflecting. This cyclic process allows students to understand phenomena in a natural setting as the concepts are better contextualised. Such lessons can also be more fun and meaningful for students.

**Relevance to the English Curriculum**

In learning new vocabulary words, students may observe how a word is spelt and the various letters that are arranged to make meaning. Through painstaking practice and repetition, it is possible to memorise how to spell the word. This approach might be familiar to many, but the challenge of contextualisation is still evident. What does the word mean and how can one use it?

If learners are given opportunities to learn what the words mean and how they are used in daily life, learners could construct their own understanding of the words. There would then be a greater likelihood of them remembering and applying their understanding in their lives. Indeed, it was found that with explicit teaching of inductive reasoning, one’s language processing skills can improve (Marx, 2009).
WHY

Using it in the primary Mathematics class

More than a hundred years ago, Stamper (1909) wrote that learners are rarely given the opportunity to engage in practical activities when learning geometrical concepts. With excessively deductive and possibly didactic approaches, learners may lose valuable opportunities to create knowledge on their own.

Today, with the advent of advanced technological programs, learners can create their own digital objects to play with and observe mathematical concepts. One such program is GeoGebra which allows users to create a wide array of shapes, measure them and discover mathematical principles through inductive reasoning. While such lessons may require more time and planning, they provide an invaluable context for learners to form their own understandings of abstract maths concepts.

Using it in the primary English class

Flashcards can be effective for introducing new vocabulary words to primary school students, especially in the lower primary levels. Associating words with relevant and meaningful pictures can illustrate the words very well.

With technology, teachers are no longer limited to using flashcards, as pictures can be easily included in slides to illustrate words in the context of a larger picture.

With numerous online tools that are readily available, inductive reasoning can be introduced to encourage students to gain greater ownership over their learning and create their own understanding of new vocabulary words.

Useful resources

1. Teaching Beginning Reading and Writing with the Picture Word Inductive Model (http://www.ascd.org/publications/books/199025.aspx)

HOW

There is a wide range of pedagogical approaches which make use of inductive reasoning. We will focus on two common approaches here: inquiry-based learning and problem-based learning (for more approaches, please see Prince & Felder, 2007).

Inquiry-based learning

In inquiry-based activities, learners are provided with a scenario or challenge. They are given information that is sufficient to pique their interest but insufficient to arrive at a conclusion. As such, learners are compelled to inquire about relevant information in the right places. In some sense, almost all inductive methods may be categorised as inquiry-based approaches. In designing these activities, the key areas to note are the amount of resources provided, the structure of the task itself and whether the task is ill-defined or clearly laid out. Depending on the structure and resources provided, these activities might meet with varying levels of inertia to learn. For example, ill-defined problems with minimal resources provided could discourage students who require more learning support.
Problem-based learning

In this approach, students are given an ill-structured challenge. The central problem is usually not immediately clear, although the effects of the problem may be. Such problem-based activities have potential to mimic authentic situations in real life and involve students using resources in the same way that actual practitioners would. These activities would also provide opportunities for students to experience challenges similar to those that actual practitioners face. For example, a problem-based activity might require students to collaborate with each other with roles and responsibilities clearly distributed and with limited resources.

In both inquiry-based learning and problem-based learning, teachers might find it useful to consider the following questions as they design and facilitate a lesson based on inductive reasoning:

1) Do students have the prior knowledge to understand the problem at hand?
2) Do they have the necessary resources to design an experiment to form and test a hypothesis?
3) Do students have the necessary knowledge to conduct the experiment and collate the necessary data?
4) Do students have the opportunity to iterate their experiments and refine their hypotheses?
5) Do you have sufficient time and resources to design a lesson based on inductive reasoning?

Inductive reasoning can allow greater contextualisation of a problem and promote better understanding of abstract concepts as learners begin with empirical observations to build a hypothesis. This allows students to hone their ability to manage their own learning and increase ownership of their progress. In short, inductive reasoning for learning provides powerful opportunities for students to become better learners.

To this end, technology can be used to design learning environments where learners can make concrete observations and record these analyses appropriately. In the following section, we explore how one teacher skilfully utilised inductive techniques to facilitate students’ learning of mathematics concepts. In the process, technology was used to enable clear observations to be made in deriving a mathematical formula. Through an inquiry-based task, students developed their own understanding of how the area of a triangle could be calculated. The process was both fun and meaningful!

REFERENCES:


FOR MORE INFORMATION

Contact: MOE_ETD_Edulab@moe.gov.sg
LESSON PLAN

Duration: 2 Periods  
Level: Primary 5  
Subject: Mathematics  
Topic: Area of Triangle

LEARNING OBJECTIVES:
At the end of the lesson, students should be able to:

• State that the area of a triangle is half of its related rectangle
• State that the base and height of a triangle is the length and breadth of its corresponding rectangle

MATERIALS AND RESOURCES REQUIRED FOR LESSON:

<table>
<thead>
<tr>
<th>ICT</th>
<th>Materials and Resources</th>
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</thead>
<tbody>
<tr>
<td>• PowerPoint slides</td>
<td>• Teacher’s prepared worksheets</td>
</tr>
<tr>
<td>• Visualiser</td>
<td>• Triangles on blue paper</td>
</tr>
<tr>
<td>• Internet connection for TitanPad</td>
<td>• Triangles on red paper</td>
</tr>
<tr>
<td></td>
<td>• Corresponding rectangles for both blue and red triangles</td>
</tr>
</tbody>
</table>

LESSON PROCEDURES AND PEDAGOGY:

<table>
<thead>
<tr>
<th>Time</th>
<th>Procedures</th>
<th>Pedagogy</th>
<th>Materials &amp; Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mins</td>
<td><strong>Tuning-In</strong></td>
<td>Reflective</td>
<td>PowerPoint slide</td>
</tr>
</tbody>
</table>
|        | Teacher shares slides showing a rectangle and triangles within it. Teacher asks students if they recall how to calculate the area of a rectangle.  
<p>|        | Teacher explains the objective of the following activity which involves students learning how to calculate the area of a triangle. |</p>
<table>
<thead>
<tr>
<th>45 mins</th>
<th><strong>Lesson Development round of game play</strong></th>
<th>Cooperative learning</th>
<th>Teacher’s prepared worksheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher distributes 3 pieces of paper-&lt;br&gt;• 1 red and 1 blue paper that have the same set of triangles printed on it&lt;br&gt;• The 3rd piece is a worksheet that students need to complete</td>
<td>Inquiry-based learning</td>
<td>Visualiser</td>
<td></td>
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<tr>
<td>Students first measure and find the area of rectangles A, B, C and D on the worksheet.</td>
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<tr>
<td>Teacher demonstrates how to cut the 2 triangles A (one from the red paper, one from the blue paper) and show that the 2 triangles can fit into rectangle A.</td>
<td></td>
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<tr>
<td>In pairs, students try to complete triangles and rectangles B, C and D.</td>
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<tr>
<td>Triangles C and D are more challenging so teacher should keep a lookout for students having challenges.</td>
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<tr>
<td>Students are to complete the table.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>10 mins</th>
<th><strong>Lesson Closure</strong></th>
<th>Experiential learning</th>
<th>Teacher’s prepared worksheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher asks questions that guide students to arrive at the formula for calculating the area of a triangle. This is actually half the area of its related rectangle since 2 triangles make up one rectangle.</td>
<td>Extension of learning</td>
<td>TitanPad</td>
<td></td>
</tr>
<tr>
<td>Teacher links area of a triangle to half the area of its corresponding rectangle, i.e. 1/2 x length x breadth of the rectangle.</td>
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<tr>
<td>Teacher invites students to undertake a challenge online using the TitanPad platform. Students may discuss the solutions and provide suggestions to the challenge.</td>
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</tbody>
</table>
Elaine had always believed that learning was not just about teachers covering the curriculum but was also about students discovering relationships between concepts and uncovering knowledge on their own. Her passion was Math and she was constantly exploring new ways of inspiring students to learn mathematical concepts in engaging and meaningful ways. The next topic was calculating the area of triangles, a challenging one for 11-year olds.

Having spent the year with her students, Elaine knew their interests and preferred learning styles. Many enjoyed tactile activities like origami and artwork. So Elaine decided to approach the topic using an inductive approach where students would learn by observing and experimenting.

“All right everyone, we’ve got a great activity lined up. Did everyone bring their scissors and sticky tubes?” asked Elaine. “YES!” replied her class excitedly. Before distributing the sheets of paper with carefully labelled triangles printed on them, Elaine went through a quick recap of the concepts from the previous lesson. She asked the class a few questions to test their understanding and they responded by writing their answers on a mini-whiteboard that each student had. Satisfied that they remembered how to calculate the area of a rectangle, Elaine moved on to the main activity. But before that, she focussed their attention with a simple question: Observe the area of the related rectangles and triangles.

“Each of you should have a blue and a red piece of paper with four triangles on each paper. On the plain piece of paper, there are rectangles printed on them. Cut out the triangles and observe how they are related to the rectangles.”

The students got down to work quickly. They meticulously cut out each triangle and tried to fit them onto the rectangles. The first two rectangles were easy as the triangles fit perfectly into them.

“Teacher, the triangles are half the area of the rectangle!” said one group of students gleefully. Another group of students were frowning as they could not find a way to fit some triangles into the rectangles.

“That’s great! Now what can you tell about the third and fourth rectangle?” asked Elaine as she saw some students’ puzzled looks in trying to fit the remaining triangles into the other rectangles. “Wow, triangle C and rectangle C are really difficult. Seriously! It doesn’t look like it’s half the rectangle?” asked one student.

“Keep trying to see the pattern everyone and remember to observe the area of the related rectangles and triangles,” Elaine reminded the class. It was easy to get distracted with all the cutting and pasting so this was a timely prompt for everyone.

After a while, some students raised their hands triumphantly and asked Elaine to check their answers by their work stations. They pointed out that the triangles, while they didn’t seem to occupy half the rectangles at the first glance, had the same breadth and height as the rectangle. Elaine was visibly delighted that the students had discovered this relationship on their own!
After all the students had completed the hands-on activity, Elaine threw another challenge to her students but this time, through a visual slide.

“Take a look at this green rectangle, what do you notice about the blue triangle within it?” Elaine invited some students up to the whiteboard to share their answers. Some students volunteered to solve the problem and ran up to the board to draw on the projected images. They carefully traced out the shape of the triangle and divided the rectangle into different parts. Then, they shaded the respective segments to show that the triangle was indeed half of the rectangle. Once again, the students pointed out that the base of the triangle was similar to the length of the rectangle while the height of the triangle was the same as the rectangle’s breadth. The class also pointed out that the height of the triangle was perpendicular to its base, just as the breadth of the rectangle was perpendicular to the length.

“All right, now can anyone tell me how I can calculate the area of any triangle?” Elaine asked hopefully. “Don’t shout the answers out. Just write them down on your mini-whiteboards.”

For a few seconds, there was a dramatic pause as the students scribbled their thoughts on their own whiteboards. Soon, numerous students began waving the mini-whiteboards in the air while trying hard not to yell the answer in their excitement.

“That’s exactly right! Can you explain the formula?” asked Elaine.

A few students explained that the area of a triangle was equivalent to the base multiplied by its perpendicular height and then divided by two.

Elaine was elated that the students had arrived at the formula. But she wanted to stretch their learning further by continuing the discussion online. “That’s just fantastic everyone. But I want to continue this discussion as I’ve got more challenging examples for you to try out. Please copy the URL and log into TitanPad to continue the fun,” said Elaine, as her students started copying the web address quickly.

Just then, the bell chimed to signal the end of the lesson. Elaine looked at her class and gave a satisfied smile. She had successfully covered the lesson objective. But more importantly, her students had discovered a mathematical formula all on their own through simple but careful observations. Now they were inspired to tackle more challenges online and extend their learning further.